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RESEARCH ARTICLE

Formulation of the Optimization Problem for Unit Commitment with Price Elasticity-Based Demand Response

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Abstract

Integrating Demand Response (DR) programs into the Unit Commitment (UC) problem is a promising method to enhance the efficiency and reliability of power systems. This work introduces a new formulation that incorporates price elasticity into the UC problem using a relaxed optimization approach. Our objective is to maximize overall system performance by reducing generation costs and maximizing the utility function while accounting for how demand changes in response to electricity prices, i.e., price elasticity-based DR. The proposed model employs Mixed-Integer Linear Programming (MILP) techniques to efficiently solve the UC problem, using a linear function to model price elasticity-based DR. Our approach has demonstrated its effectiveness in achieving substantial cost reductions and improved load management, as shown through numerical simulations.

Keywords: Unit Commitment, Demand Response, Social Welfare

Nomenclature

d_t initial system demand level in hour
$\hat{d}_t$ final demand level in hour
d_t^{cr} quantity of demand curtailed in hour t
$d_{t,t'}^{Pt}$ demand quantity shifted from hour t to t'
$d_{t,t'}^{Pt}$ demand quantity shifted from hour t to t
d_t^{min} minimum bounds for $\hat{d}_t$
d_t^{max} maximum bounds for $\hat{d}_t$
$P_{t,g}^{min}$ minimum operating limit of the generator

$P_{t,g}^{max}$ maximum operating limit of the generator
MRU_n maximum ramp-up limit, unit n MW
MRD_n maximum ramp-down limit, unit n MW
MUT_g minimum up time for unit g hour t
MDT_g minimum downtime for unit g hour t
$U_{g,t}$ binary state variable for unit g hour t
$S_{g,t}$ start-up variable for unit g hour t
$h_{g,t}$ shut-down variable for unit g hour t
F social welfare function
$C_g\left(P_{t,g}\right)$ fuel cost
w designed parameter
$\gamma_{t,t'}$ upper bound of model error
σ elasticity coefficient
a_g, b_g, c_g cost function coefficient of unit g
$\varepsilon_{t,t}$ the price elasticity
$\varepsilon_{t,t'}$ the price elasticity
$U_t\left(\hat{d}_t\right)$ utility function

1. Introduction

Electricity is a significant energy source relied upon by society for various daily activities. Currently, there is a shift in consumer demand for electricity. Consumers not only want to purchase kilowatt-hours (kWh) from electric companies, but they also want to receive better services. Over time, electric companies will transition to a condition where electricity demand must be adjusted to the capacity of the electricity supply. Efforts are currently underway to regulate the consumption of electric energy by users to align with conditions on the production side, commonly known as Demand-Side Management (DSM) [1].

DSM helps efficiently utilize generation capacities and minimizes the underutilization of generation resources in the system. An important aspect of DSM is reducing peak load demand and ensuring a sustainable power supply to consumers at reduced operational costs. This is possible either by reducing load demand or by shifting the load [2]. Load shifting is one of effective DR program, as the load independent of time can be transferred to off-peak hours [3].

Electricity companies develop operational policies for the power system to overcome energy distribution problems for users. These plans, including short-term scheduling strategies, outline operational guidelines for the system over a given timeframe, ensuring effective and efficient energy provision. One of the short-term operational plans in the power system is unit commitment operation scheduling. Unit Commitment is vital to power system functioning and electrical network planning. This procedure involves identifying the optimal operation of generating units during specified periods to meet power demand at the lowest possible cost while ensuring system reliability.

The most discussed techniques for solving the unit commitment problem are priority-list schemes, Dynamic Programming (DP), and MILP. Priority-list schemes are the most popular. DP algorithms are the only ones approaching an optimum solution for large systems. MILP algorithms are just beginning to be researched and are not widely used for large-system problems [4]. UC admits a natural formulation in MILP, as demonstrated in [5] long before it was practical.

MILP has been widely used to address UC problems due to significant advances in widely available MILP solvers that use branch-and-cut techniques. Due to advances in computer hardware and algorithmic efficiency, the time required to solve a MILP has decreased by 100 million-fold in the last two decades [6]. Researchers have recently suggested employing a MILP formulation approximated by UC [7],[8],[9]. This formulation utilizes highly efficient, versatile MILP solvers to rapidly calculate high-quality answers, particularly for small to mediumsized problems. Nevertheless, larger cases may still require specialist procedures.

Recently, due to high electricity demand, integration of renewable energy sources, and changes in the dynamics of energy consumption, the power system has faced significant challenges. Among the increasing interest in an emerging strategy to modify energy consumption through consumer engagement is the adoption of demand response. Price elasticity is important in optimizing unit commitment when considering demand response. Price elasticity measures the sensitivity of changes in demand that occur in response to price changes. Understanding price elasticity can improve load management and reduce operating costs by utilizing the demand flexibility of the system operators.

Price elasticity of demand was used to model demand response, embedding responsiveness of demand in the iterative market clearing process [10]. Reference [11] presents an economic demand response model that embeds both price and incentive-responsive loads. The model utilizes flexible demand elasticity as a measure of market flexibility.

However, the research cited earlier [12],[11],[13] focuses on price-based elasticity, even in augmented demand response models used for analysis. This amounts to a failure to consider the effect of incentives or penalties that would impact elasticity. However, incorporating price elasticity-based demand response into the unit commitment process is complex. Accurately representing the connection between pricing and demand, as well as managing the intricacies of the interplay between generating units and demand response, necessitates the use of appropriate mathematical formulations. Relaxation formulations are one of the very effective ways to tackle such problems by simplifying the model and maintaining accuracy.

The proposed work focuses on designing and testing an approach to unit commitment that incorporates price elasticity-based demand response via relaxation formulations. This approach is expected to increase operations' efficiency and flexibility in handling the power system with fluctuating demands and energy price changes. The proposed approach aims to reduce the demand for electricity during peak times by giving priority or shifting the load. This way, consumers will reduce non-critical or shiftable load utilization during peak demand periods, lessening energy demand. This involves load reduction or load redistribution to enhance net usefulness on the consumer side while reducing the cost of generation and maximizing social welfare. This strategy considers the respective generating unit constraints, which include the minimum and maximum capacity, the maximum and minimum ramp rates, and the minimum uptime and downtime.

The structure of this paper is outlined as follows. Section 2 presents the fundamentals of the UC and DR Model. In Section 3, we introduce the idea and fundamental unit commitment model considering price elasticity-based demand response and simplify the difficulties of problems that common computational constraints could solve. In Section 4, we tested our proposed model with daily load data (24 hours). Finally, we summarize our findings and propose avenues for future research in Section 5.

2. Preliminaries

This section covers the preliminary foundations and background necessary to understand the basic structure of unit commitment and the demand response model. It provides the foundation for the approaches and models used in later sections.

2.1 Unit Commitment

UC is the short-term scheduling of daily or weekly electricity generation from a thermal unit to meet current and anticipated electricity demand most economically by scheduling the unit on/off for the best operating conditions of the power plant while meeting technical constraints. The scheduling used by UC requires long-term differences of several hours to days, considering physical constraints such as minimum up-down time and maximum and minimum generation limits [14]. The main motivation for UC operation is to minimize electricity generation costs during the same planning period. Let $P_{t,g}$ be the generation of generator g at hour t; then, UC is formulated as follows:

$$\min_{P} \sum_{t=1}^{T} \sum_{g=1}^{G} C_g(P_{t,g}),$$

subject to system and unit constraints where G is the number of generators and T is the period. The detailed mathematical formulation for the objective function and the constraints in this paper are presented next.

Fuel Cost

The fuel cost function of a thermal generator is given in quadratic form as follows:

$$C_g\left(P_{t,g}\right) = a_g + b_g P_{t,g} + C_g\left(P_{t,g}\right)^2. \tag{1}$$

System Constraints

The problem includes the supply-demand balance equation, i.e.,

$$\sum_{g=1}^{G} P_{t,g} = d_t. \tag{2}$$

Generation Limit Constraints

In practice, the generation units have limited fuel or are required to burn a specific amount of fuel in a given time. This can be written as

$$P_{t,g}^{min} u_{g,t} \leq P_{t,g} \leq P_{t,g}^{max} u_{g,t}. \tag{3}$$

Minimum Up-Down Time Constraints

Let $u_{g,t}$ be the binary state variable, $s_{g,t}$ be the start-up variable, and $h_{g,t}$ be the shut-down variable for unit g, hour t. Then, the restrictions can be formulated as follows [15]:

$$\sum_{i=i-MUT_g+1}^{t} s_{g,i} \leq u_{g,t}, \quad \forall g, \quad \forall t \in \left[MUT_g + 1, T\right], \tag{4}$$

$$\sum_{i=i-MDT_g+1}^{t} h_{g,i} \leq 1 - u_{g,t}, \quad \forall g, \quad \forall t \in \left[MDT_g + 1, T\right], \tag{5}$$

where MUT_g and MDT_g are the minimum on and off time, respectively.

Ramp Rate Constraints

The ramp of generation output is limited, i.e.,

$$-MRU_n \leq P_{t,g} - P_{t+1,g} \leq MRD_n, \tag{6}$$

where MRU_n and MRD_n are the maximum ramp-up and ramp-down limits, respectively.

2.2 Demand Response Model

DR gives customers a chance to actively contribute to the electrical grid by modifying their usage patterns in response to the price of electricity or other financial incentives [16],[17]. The most common changes that customers can make are *curtailment or shifting* their electricity consumption at times when it is valuable to the electricity system or to the customers themselves.

Load Curtailment

Curtailment has been employed by the system's operator to ensure the proper operation of power systems. By reducing a portion of the load, catastrophic events such as complete blackouts can be avoided. Then, the total demand $\hat{d}_t$ after the amount of energy to be curtailed d_t^{cr} can be written as follows:

$$\hat{d}_t = d_t - d_t^{cr}, \tag{7}$$

$$\hat{d}_t \geq 0, d_t^{cr} \geq 0, \tag{8}$$

where d_t is the base load.

Load Shifting

Load shifting or deferral is a widely used method in demand response, allowing consumption to be delayed or anticipated. The main limitations are technical constraints, process requirements, and the availability of unused plant capacity. From a system viewpoint, load shifting mimics the functionality of conventional storage units by reducing demand when electricity prices are high and increasing it when prices are low. The primary difference between DR shifting and storage is that DR storage, like that of electric vehicles (EVs), must always meet a specific demand, ensuring constant consumption. Let $d_{t,t'}$ be the amount of demand that is shifted from hour t to t' , where $t' \neq t$, then the amount of supplied demand due to load shifting is:

$$\hat{d}_t = d_t - \sum_{t' \neq t} d_{t,t'} + \sum_{t' \neq t} d_{t',t}. \tag{9}$$

Demand Response under Time-of-Use Pricing

In order to promote benefits for both generators and customers from economic operations, DR under time of use (TOU) pricing is considered. Demand response under TOU pricing is a mutually beneficial approach that supports economic operations for generators and cost savings for customers. By aligning energy consumption patterns with periods of lower demand and cost, TOU pricing enhances grid stability, reduces operational costs, and contributes to environmental sustainability. A core principle of this program is that the total load increased and the total load decreased over the entire time period must be balanced and equal. The limitation can be written as follows [18]:

$$\sum_{t=1}^{T} \hat{d}_t = \sum_{t=1}^{T} d_t, \tag{10}$$

$$d_t^{min} \leq \hat{d}_t \leq d_t^{max}, \forall t. \tag{11}$$

The developed model will ensure total energy consumption over the T-hour period. It achieves this by allocating the flexible portion of the demand to times when prices are lowest, all while adhering to demand capacity constraints. The value of (11) indicates the upper and lower bounds of DR. Based on the method adopted from [19], the maximum and minimum demand values are defined as $d_t^{max} = (1 + \sigma)d_t$ and $d_t^{min} = (1 - \sigma)d_t$, where σ is an elasticity coefficient selected within the interval [0.02, 0.05].

2.3 DR with Cross Price Elasticity

In this paper, we concentrate on a DR program where the demand side reduces or shifts a portion of its electricity usage across different hours in response to a price signal. For example, if the price of electricity during peak hours increases by 20.

The price elasticity in the hour t corresponding to the electricity consumption of t itself and is denoted by $\varepsilon_{t,t}$, where $t \in \{1, \ldots, T\}$. Furthermore, cross-price elasticity corresponding to the demand in hour t and the price in hour t' is represented by $\varepsilon_{t,t'}$ where $t \in \{1, \ldots, T\}$ and $t' \in \{1, \ldots, T\} \setminus t$.

Consider E as the $T \times T$ matrix that includes the price elasticities for different hours. The diagonal elements $\varepsilon_{t,t}$ indicate the price elasticity within the same hour, while the off-diagonal elements $\varepsilon_{t,t}$ indicate the cross-price elasticity between different hours t and t' where $t \neq t'$. To define the utility functions and constraints on the demand side for each hour t (assuming $T = 24$ hours), we establish the relationship between the elements of E and the price-responsive demand variable $\hat{d}_t$.

The resulting amount of demand after the DR $\hat{d}_t$ can be defined as a function of the price vector $\mathbf{P} = [P_1, \ldots, P_T]^{\dagger}$ (where † denotes the transpose), i.e., $\hat{d}_t(\mathbf{P})$, where

$$\mathbf{P} = P_0(\mathbf{1}_T - E^{-1}\mathbf{1}_T) + P_0E^{-1}\left(\frac{\hat{d}_1}{d_1}, \ldots, \frac{\hat{d}_T}{d_T}\right)^{\dagger}.$$

Here, E^{-1} is the inverse of the matrix E. Each element of $\mathbf{P}$ represents the inverse demand function or the price for each corresponding hour. See [20] for details.

Let the curtailed demand d_t^{cr} in hour t be a response to the price in hour t (denoted by P_t). Furthermore, $d_{t,t'}^{p_t}, t' \neq t$, is the decision variable to represent the demand quantity shifted from hour t to t' in response to the price in hour t, and d $d_{t,t'}^{p_{t'}}$ (where t' , t) is the new decision variable to represent the demand quantity shifted from hour t to t' in response to the price in hour t' . One can observe that

$$d_{t,t'}^{P_{t'}} = -d_{t',t}^{P_{t'}}, \quad \forall t,\ t' \neq t. \tag{12}$$

It is essential to understand that demand curtailment within any specific hour t is directly influenced only by the price change in that same hour. In contrast, demand shifting between two different hours t and t' is driven by the price changes occurring in both hours t and t' . This means that while curtailment reacts to immediate price fluctuations, shifting considers the relative price differences between the two hours. The demand side, considering curtailment and shifting based on cross-elasticity, is given as follows [20]:

$$\hat{d}_t = d_t - d_t^{cr} - \sum_{t' \neq t} d_{t,t'}^{P_t} - \sum_{t' \neq t} d_{t,t'}^{Pt'} \forall t, \tag{13}$$

$$d_{t,t'}^{P_t} + \frac{\varepsilon_{t,t}d_{t'}}{\varepsilon_{t,t}d_t}\left(d_t^{cr} + \sum_{t'' \neq t} d_{t,t'}^{Pt}\right) = 0 \quad \forall t, \forall t' \neq t. \tag{14}$$

It is important to mention that $d_{t,t'}^{P_t}$ might have positive or negative values at different hours. A positive value indicates that the load during that hour has been redistributed to other hours, whereas a negative value indicates that the load from other hours has been moved to that hour.

3. Unit Commitment with Demand Response Models

The DR models are predominantly formulated to correctly model the optimal planning and operation of power and energy systems (including markets). In this section, the aggregated UC problem considering the DR in the form is discussed. First, social welfare of the demand response program is discussed. Social welfare is defined as the aggregate utility of the demand side's overall hours minus the total penalties incurred by the demand side and the total generation costs of the supply side. Then, the proposed optimization problem is presented.

3.1 Social Welfare in DR

In the context of demand response, social welfare refers to the overall well-being and economic efficiency achieved by optimizing the electricity consumption and production in response to price signals. The social welfare function in this scenario evaluates the aggregate utility of electricity consumption adjusted for the costs and benefits associated with demand response activities.

A social welfare function assigns a ranking to different social states, indicating the desirability of each state based on the well-being of individuals in society. In the context of demand response, this function considers the utility gained by consumers from electricity usage and the costs incurred by producers. The goal is to find an optimal balance that maximizes the overall utility while minimizing the costs. The general form of the social welfare function can be expressed as:

$$F = \sum_{t=1}^{T} \left(U_t\left(\hat{d}_t\right) - \sum_{g=1}^{G} C_g\left(P_{t,g}\right) \right). \tag{15}$$

The utility function $U_t\left(\hat{d}_t\right)$ reflects the satisfaction or benefit that consumers derive from consuming electricity. This can be modeled as a concave function, indicating that the marginal utility of consumption decreases as consumption increases:

$$U_t\left(\hat{d}_t\right) = M_t \hat{d}_t, \tag{16}$$

where $M_t \geq 0$ is parameters that define the utility function.

3.2 Proposed Optimization Problem

This objective is subject to the constraints imposed by both the demand and supply sides. In other words, the problem can be formulated as follows:

$$\max_{P,\hat{d},\gamma} \quad F - w \sum_{t=1}^{T} \sum_{t' \neq t}^{T} \gamma_{t,t'} \tag{17}$$

subject to

$$d_{t,t'}^{P_t} + \frac{\varepsilon_{t,t'} d_{t'}}{\varepsilon_{t,t} d_t} \left(d_t^{cr} + \sum_{t'' \neq t} d_{t,t''}^{P_t} \right) \leq \gamma_{t,t'}, \quad \forall t, t \neq t \tag{18}$$

$$\sum_{t=1}^{T} \hat{d}_t = \sum_{t=1}^{T} d_t - \sum_{t=1}^{T} d_t^{cr} \tag{19}$$

$$\gamma_{t,t'} \geq 0, \forall t, t \neq t, (1) - (6), (8), (11) - (13),$$

where F is defined in (15) and $w > 0$ is a design parameter. The objective function is formulated such that it minimizes the social welfare F while trying to minimize the total $w\gamma_{t,t'}$ for all t and $t \neq t$. Note that the $\gamma_{t,t'}$ can be seen as an upper bound of model error of equation (14). In other words, we would like to put the DR-based price elasticity model into consideration. However, instead of using (14) as a constraint, we use its relaxed problem to increase the feasibility region of the proposed optimization problem. Furthermore, the problem incorporates the constraints from the UC, i.e., (1)-(6), and the DR-program (11)-(13). The constraint (19) is incorporated to consider (10) when curtailed demand is considered. In the next section, the proposed optimization is verified using numerical simulations.

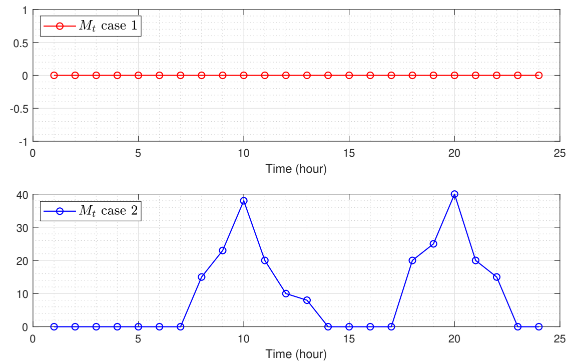

Figure 1. Parameter M_t for utility function (16)

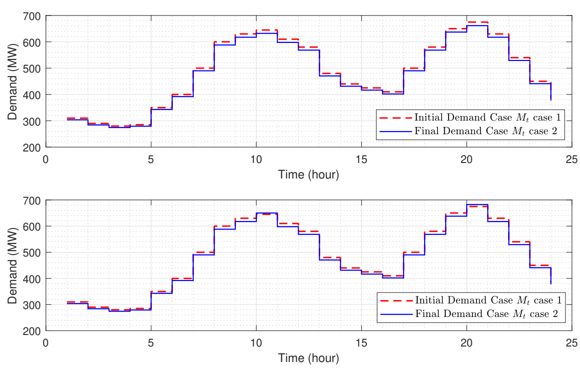

Figure 2. Comparison of base demand and final demand (after DR) for Case 1 and 2

4. Numerical Simulations

This section conducts numerical simulations using the system parameter data from [4] to demonstrate the efficiency of the proposed UC with DR program. To be precise, we aim to understand of how demand response can provide flexibility to the grid and its potential to reduce production costs and increase overall system efficiency. The proposed optimization problem is solved using parameter design $w = 10$. The simulations were performed on a PC with AMD Dual Core A9-9420 3.6 GHz CPU and 4 GB memory. All algorithms are implemented on MATLAB and programmed using YALMIP environment [21]. The solver used is GUROBI 11.0 [22], with the optimality gap is set to be 0.1.

4.1 The Impact of The Utility Function

This study assessed social welfare using two different parameters for the utility function M_t, as shown in Figure 1. We identified two scenarios: Case 1 and Case 2. In Case 1, M_t is zero for all t, indicating that the function does not aim to maximize electricity consumption. Conversely, in Case 2, M_t is increased during two specific periods. The resulting final demand is depicted in Figure 2. In Case 1, it is evident that during peak load times at $t = 10$ and $t = 20$, the final demand is lower than the initial demand, reflecting reduced electricity consumption during these periods. In contrast, in Case 2, the final demand at $t = 10$ and $t = 20$ closely matches the initial demand, as the proposed method also seeks to maximize electricity consumption by customers.

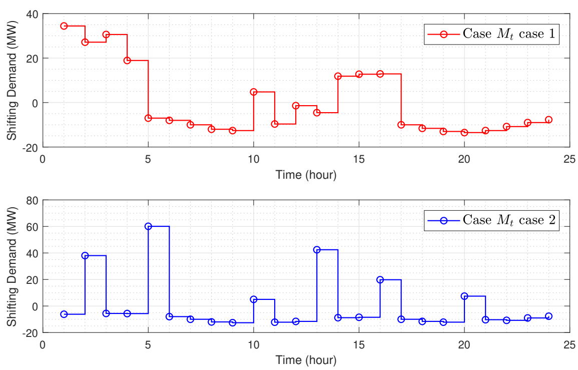

Figure 3. Shifted demand (MW) for Case 1 and 2

Additionally, during other periods, the final demand is a result of both shifted and curtailed demand, as shown in Figures 3 and 4. Despite $M_{10} \neq 0$ and $M_{20} \neq 0$ in Case 2, the shifted and curtailed demand during these periods are not zero. This is because the proposed method uses the penalty function $w\gamma_{t,t'}$ to ensure that the resulting demand reflects the customers' behavior in response to electricity prices. Table 1, presents the sum of $\gamma_{t,t'}$ over all time periods T. This sum represents the error associated with the proposed method under a price elasticity based DR model. This shows that, despite using a relaxed version of the problem, the resulting error is still

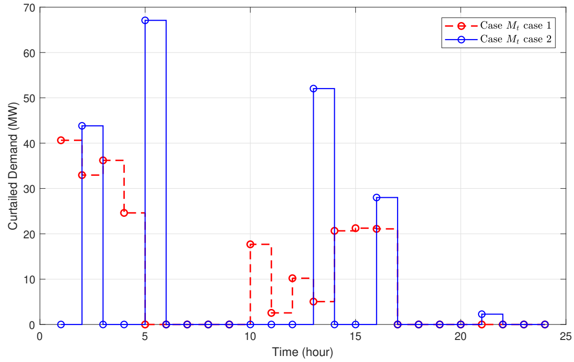

Figure 4. Curtailed demand (MW) for Case 1 and 2

maintained at a small level, indicating the effectiveness and precision of the proposed method in modeling demand response while adhering to price elasticity principles.

4.2 The Impact of The Demand Response Model on Unit Commitment

Demand Response could be defined as a type of negative generation in the unit commitment model. Hence, this section aims to assess the effects that demand response may have on system capacity. To this end, the proposed UC with DR problem is solved using parameter M_t shown in Figure 5.

Table 1. Total value of upper bound $\gamma_{t,t'}$ using different utility function.

Case	Total $\gamma_{t,t'}$
Case 1	1.33926
Case 2	1.9.8448

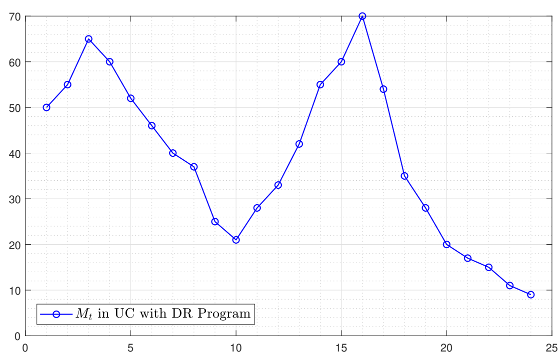

Figure 5. Parameter M_t for utility function (16)

Figure 6 shows the difference between the initial request and the final request which means that the demand response program has affected energy consumption. This change can be caused by a decrease in consumption by consumers during peak demand times or from a shift of consumption to another period. Figure 7 compares the power generated with and without DR program for 24 hours. Focusing on the period from 1 a.m. to 6 a.m. and then 2 p.m. to 5 p.m., the total energy produced with DR is higher than the power produced without DR because the constant economic value given is sufficient to motivate consumers to transfer their energy consumption to the clock so it requires power to be generated.

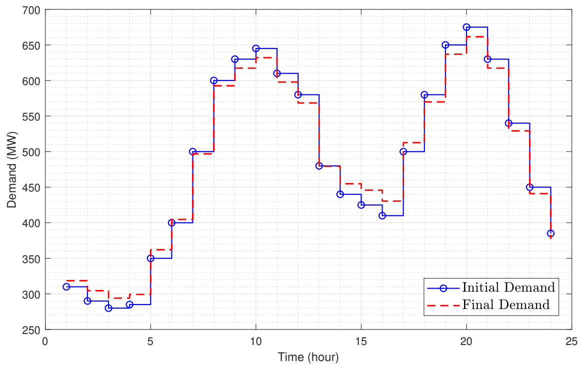

Figure 6. Comparison of initial and final demand with DR program

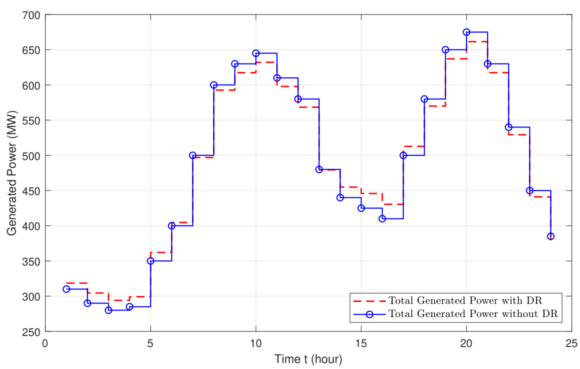

Figure 7. Total generated power when UC is solved with and without DR program

In addition, as shown in Table 2, implementing demand response in the power system has reduced the production costs. The impact DR has on a reduction in operational costs is effected by realizing efficient demand adjustments that directly reduce the peak load on the system and, subsequently, the expensive burden of additional generation on the system. This indicates that DR is a good measure to

Table 2. Total generation cost with and without demand response (DR)

Case	Generation Cost ($/MW)
With DR	2.5925e+05
Without DR	2.6440e+05

increase operational efficiency and, in turn, decrease the costs associated with energy management. These forms of benefits are important in achieving better energy management, with more economically planned power generation capacity, which is going to ensure a more stable and sustainable electricity supply.

5. Conclusion

This paper develops an optimization approach for unit commitment using a relaxed formulation to incorporate price elasticity-based demand response. The objective is to maximize social welfare, defined as the utility function minus the generation cost. Incorporating demand response programs in unit commitment can significantly reduce peak load and better manage energy consumption during non-peak periods. Future research should explore using price elasticity-driven demand response strategies to further enhance the integration of renewable energy into the electricity grid. Developing adaptable models of price elasticity is crucial for accurately capturing the diverse ways consumers react to varying situations and time periods, which may improve the flexibility and efficiency of the power system in the future.

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